

General Instructions :

The question paper is divided into **four** sections.

- 1) **Section A** - Q. No. 1 contains **eight** multiple choice type of questions, each carrying Two marks.
Q. No. 2 contains **four** very short answer type questions, each carrying One mark.
- 2) **Section B** - Q. No. 3 to Q. No. 14 contain **twelve** short answer type questions each, carrying Two marks. (Attempt any **eight**).
- 3) **Section C** - Q. No. 15 to Q. No. 26 contain **twelve** short answer type questions, each carrying Three marks. (Attempt any **eight**).
- 4) **Section D** - Q. No. 27 to Q. No. 34 contain **eight** long answer type questions, each carrying Four marks. (Attempt any **five**).
- 5) Use of log table is allowed. Use of calculator is not allowed.
- 6) Figures to the right indicate full marks.
- 7) Use of graph paper is not necessary. Only rough sketch of graph is expected.
- 8) For each multiple choice type of question, it is mandatory to write the correct answer along with its alphabet. e.g. (a) / (b) / (c) / (d) No mark (s) shall be given if **ONLY** the correct answer or the alphabet of the correct answer is written. Only the first attempt will be considered for evaluation.
- 9) Start answer to each section on a new page.

SECTION - A

Q. 1 Select and write the correct answer for the following multiple choice type of questions. 16

- i) If $(p \wedge \sim q) \rightarrow (\sim p \vee r)$ is a false statement, then the respective truth values of p , q and r are _____

a) T, F, F b) F, T, T c) T, T, T d) F, F, F

- ii) If $A = \begin{bmatrix} 0 & 1 & 2 \\ 1 & 2 & 3 \\ 3 & 1 & 1 \end{bmatrix}$, then the sum of all the diagonal entries of A^{-1} is _____

a) 2 b) 3 c) -3 d) 4

- iii) If $\tan \theta = \frac{-1}{\sqrt{3}}$, $\sin \theta = \frac{1}{2}$, $\cos \theta = \frac{-\sqrt{3}}{2}$ then the principal value of θ will be

a) $\frac{\pi}{6}$ b) $\frac{5\pi}{6}$ c) $\frac{7\pi}{6}$ d) $\frac{11\pi}{6}$

- iv) Two line represented by the equation $x^2 + xy + y^2 = 0$ are _____

a) Coincident b) Parallel
c) Mutually perpendicular d) Imaginary

- v) $y = \cos^2 \frac{3x}{2} - \sin^2 \frac{3x}{2}$ then $\frac{d^2y}{dx^2}$ is _____
- a) $-3\sqrt{1-y^2}$ b) $9y$ c) $-9y$ d) $3\sqrt{1-y^2}$
- vi) A particle moves according to the law $s = t^3 - 6t^2 + 9t + 25$. The displacement of the particle at the time when its acceleration is zero is. _____
- a) -27 units b) -9 units c) 27 units d) 9 units
- vii) For a random variable X if $E(X) = 5$ and $\text{Var}(X) = 6$, then $E(X^2)$ is equal to _____
- a) 19 b) 31 c) 61 d) 11
- viii) If $X \sim B(4, p)$ and $P(X=0) = \frac{16}{81}$ then $P(X=4) =$ _____
- a) $\frac{1}{27}$ b) $\frac{1}{16}$ c) $\frac{1}{81}$ d) $\frac{1}{8}$

Q. 2 Answer the following questions.

- 1) Write the contrapositive of the statement.
"If you are born in India, then you are a citizen of India."
- 2) Evaluate $\tan^{-1} \sqrt{3} - \cot^{-1}(-\sqrt{3})$
- 3) If $y = \log_x a$ find $\frac{dy}{dx}$.
- 4) Find a if $f(x) = x^3 - 3x$ has minimum value at $x = a$.

SECTION - B

Attempt any Eight of the following questions.

Q. 3 Write the negations of the following statement.

- i) $\forall x \in N, 2x + 1$ is odd.
- ii) some triangles are equilateral triangle.

Q. 4 If A is a matrix such that $A(\text{adj } A) = \begin{bmatrix} 20 & 0 \\ 0 & 20 \end{bmatrix}$ find $|A|$.

Q. 5 In ΔABC , If $a = 18$, $b = 24$ and $c = 30$ then find the values of $\cos \frac{A}{2}$

Q. 6 Prove that : $2 \tan^{-1} \frac{1}{3} = \tan^{-1} \frac{3}{4}$.

Q. 7 Find K if the slope of one of the lines is given by $3x^2 + 4xy + Ky^2 = 0$ is 1 .

Q. 8 Find the graphical solution for the linear in equations.

$$2x + y \geq 2, \quad x - y \leq 1$$

Q. 9 If $y = \tan^{-1} \left(\frac{1 - \cos 3x}{\sin 3x} \right)$, find $\frac{dy}{dx}$

Q. 10 The edge of a cube is decreasing at the rate of 0.6 cm/sec . Find the rate at which its volume is decreasing when the edge of the cube is 2 cm .

- Q. 11 Find the values of x for which $f(x) = x + \frac{25}{x}$ is strictly decreasing.
- Q. 12 If $y = (\sin x)^x$ find $\frac{dy}{dx}$.
- Q. 13 Find K if the probability mass function of random variable X is given by
- $$P(x) = Kx^2 \quad x = 1, 2, 3, 4$$
- $$= 0 \quad \text{otherwise}$$
- Hence find $F(2)$
- Q. 14 Given $X \sim B(n, p)$
if $n = 10$, $E(X) = 8$, find $\text{Var}(X)$.

SECTION - C

Attempt any Eight of the following.

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- Q. 15 Construct the switching circuit and write the input-output table for $(\sim p \wedge q) \vee (p \wedge \sim q)$
- Q. 16 If $A = \begin{bmatrix} 2 & -3 \\ -1 & 2 \end{bmatrix}$ find A^{-1} using is elementary transformation.
- Q. 17 With the usual notations, prove that
- $$2 \left[a \sin^2 \frac{C}{2} + c \sin^2 \frac{A}{2} \right] = a - b + c$$
- Q. 18 Find the general solution for the equation $\sin \theta = \tan \theta$.
- Q. 19 Prove that the acute angle between the lines represented by the equation.
- $$ax^2 + 2hxy + by^2 = 0 \text{ is } \tan \theta = \left| \frac{2 \sqrt{h^2 - ab}}{a + b} \right|$$
- Q. 20 In the equation $x^2 - y^2 + kx + 1 = 0$ represents pair of lines. Find k , also find their point of intersection.
- Q. 21 If $x = f(t)$ and $y = g(t)$ are differentiable function of t , then prove that
- $$\frac{dy}{dx} = \frac{dy/dt}{dx/dt}, \text{ if } \frac{dx}{dt} \neq 0.$$
- Q. 22 If $y = e^{m \cos^{-1} x}$, prove that
- $$(1 - x^2) \frac{d^2 y}{dx^2} - x \frac{dy}{dx} - m^2 y = 0$$
- Q. 23 If Rolle's theorem holds for the function $f(x) = (x - 2) \log x$, $x \in [1, 2]$, show that the equation $x \log x = 2 - x$ is satisfied by atleast one value of x in $(1, 2)$.

- Q. 24 Show that among rectangles of given area, the square has least perimeter.
- Q. 25 The following is the pdf of continuous r.v. X .
- $$f(x) = \frac{x}{8}, \quad 0 < x < 4$$
- $$= 0, \quad \text{otherwise}$$
- i) Find the expression for c.d.f of X .
- ii) Hence find $F(5)$.
- Q. 26 Five cards are drawn successively with replacement from a well-shuffled of 52 cards find the probability that atleast one card is a spade.

SECTION - D

Attempt any Five of the following questions.

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- Q. 27 Without using truth table, prove that.
 $p \leftrightarrow q \equiv (p \wedge q) \vee (\sim p \wedge \sim q)$
- Q. 28 Solve the following system of equations by the method of Reduction.
 $2x - y + z = 1$, $x + 2y + 3z = 8$, $3x + y - 4z = 1$
- Q. 29 If A, B, C are the measures of the angles of a ΔABC and a, b , and c are the lengths of the sides opposite to the angles A, B and C respectively then prove that.

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C} = 2R, \text{ } R \text{ is the circum radius of } \Delta ABC.$$
- Q. 30 Solve the following LPP by graphical method.
 $2x + 3y \geq 15$, $2x + y \geq 7$
 $y \geq 2$, $x \geq 0$, $y \geq 0$
- Q. 31 If $\sqrt{\frac{x}{y}} + \sqrt{\frac{y}{x}} = 6$, Prove that $\frac{dy}{dx} = \frac{y}{x}$
- Q. 32 Find approximate value of $\log_{10}(1016)$ given that $\log_{10}e = 0.4343$.
- Q. 33 A player tosses 2 fair coins. He wins ₹ 5 if 2 heads appear, ₹ 2 if 1 head appears and ₹ 1 if no head appears. Find his expected winning amount of variance of winning amount.
- Q. 34 There are 5 % defective items in a large bulk of items. What is the probability that a sample of 10 items will include not more than one defective item.
