

General Instructions :-**The question paper is divided into Four section.**

- Section - A : Q. No. 1 contains Eight multiple choice type of questions carrying two mark each.
Q. No. 2 contains four very short answer type of questions carrying One mark each.
 - Section - B : Q. No. 3 to Q. No. 14 contains Twelve short answer type of questions carrying two marks each. (Attempt any Eight)
 - Section - C : Q. No. 15 to Q. No. 26 contains Twelve short answer type of questions carrying Three marks each. (Attempt any Eight)
 - Section - D : Q. No. 27 to Q. No. 34 contains Eight long answer type questions carrying Four mark each. (Attempt any Five)
 - Use of logarithmic table is allowed. Use of calculator not allowed.
 - Figures to the right indicate full marks.
 - Use of graph paper is not necessary only rough sketch is expected.
 - For each MCQ, correct answer must be written along with its alphabet. Eg. a) b) c)..... d)
- Only first attempt will be considered for evaluation.
- Start answer to each section on new page.

Section A**Q.1. Select and write the correct answer for the following multiple choice type of questions.****16**

- The negation of inverse of $\sim p \rightarrow q$ is _____
a) $q \wedge p$ b) $\sim p \wedge \sim q$ c) $\sim p \wedge q$ d) $\sim q \rightarrow \sim p$
- If A is square matrix of order 2 and $A(\text{adj } A) = \begin{bmatrix} 10 & 0 \\ 0 & 10 \end{bmatrix}$ then $|A| =$ _____
a) 10 b) 100 c) 20 d) 0
- The polar coordinates of a point whose Cartesian coordinates are $\left(-\frac{1}{\sqrt{2}}, -\frac{1}{\sqrt{2}}\right)$ are _____
a) $\left(1, \frac{\pi^c}{4}\right)$ b) $\left(1, \frac{5\pi^c}{4}\right)$ c) $\left(\sqrt{2}, \frac{\pi^c}{4}\right)$ d) $(\sqrt{2}, 200^\circ)$
- The measure of angle between the two line $(3 + 2\sqrt{3})x^2 - 2xy - y^2 = 0$ is _____
a) $\frac{\pi}{2}$ b) $\frac{\pi}{3}$ c) $\frac{\pi}{4}$ d) $\frac{\pi}{6}$

5) If $y = \sin x^{\sin x^{\sin x \dots}}$ then $\frac{dy}{dx} = \underline{\hspace{2cm}}$

a) $\frac{y^2 \cot x}{\frac{1}{y} + y \log(\sin x)}$

b) $\frac{y^2 \tan x}{\frac{1}{y} - y \log(\sin x)}$

c) $\frac{y^2 \cot x}{1 - \log(\sin x)}$

d) $\frac{y \cot x}{\frac{1}{y} - \log(\sin x)}$

6) The function $f(x) = x^3 - 3x^2 + 3x - 100, x \in R$ is

- a) neither increasing nor decreasing
- b) increasing and decreasing
- c) decreasing
- d) increasing

7) The p.d.f of a continuous r.v X is given by $f(x) = Kx, 0 < x < 4$
 $= 0$ otherwise

then the value of c.d.f $F(4) = \underline{\hspace{2cm}}$

a) $\frac{1}{3}$

b) 1

c) $\frac{2}{3}$

d) 0

8) If $\frac{d}{dx}f(x) = f(x)$, then

a) $f(x) = x$

b) $f(x) = e^x$

c) $f(x) = 0$

d) $f(x) = C, C = \text{constant}$

Q.2. Answer the following questions.

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- 1) Write the Range of inverse secant function.
- 2) Write the values of x, y, z if

$$\begin{bmatrix} -1 & 0 & 0 \\ 0 & 0 & -1 \\ 0 & -1 & 0 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 4 \\ 0 \\ 2 \end{bmatrix}$$

3) If $x = t^2$ and $y = t^3$, find $\frac{dy}{dx}$

4) Find the approximate value of $f(2.01)$ if $f(2) = 27, f'(2) = 40$ and $h = 0.01$

Section B

Attempt any Eight of the following questions.

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Q.3. Write the inverse and contrapositive of the following statement :

If the function is differentiate then it is continuous.

Q.4. If $A = \begin{bmatrix} 3 & -1 \\ 4 & 2 \end{bmatrix}, B = \begin{bmatrix} 2 \\ -1 \end{bmatrix}$ find X such that $AX = B$

Q.5. Find the principal solution of $\sqrt{3} \operatorname{cosec} x + 2 = 0$

- Q.6. If $2 \sin^{-1} \frac{3}{7} = \cos^{-1} \beta$, then find the value of β .
- Q.7. If $ax^2 + 2hxy + by^2 = 0$ represents a pair of lines and $h^2 = ab \neq 0$, then find the ratio of their slopes.
- Q.8. Solve the following equations graphically and write the corner points of the feasible region :
 $3x + y \leq 15$; $3x + 4y \leq 24$; $x \geq 0, y \geq 0$
- Q.9. If $y = e^{ax}$, show that $x \frac{dy}{dx} = y \log y$.
- Q.10. Find the equation of tangent to the curve $y = 2x^3 - x^2 + 2$ at $(\frac{1}{2}, 2)$.
- Q.11. Differentiate $\log(1 + x^2)$ w. r. t $\tan^{-1}x$.
- Q.12. Obtain the probability distribution of the number of sixes in two tosses of a fair die.
- Q.13. Given that $X \sim B(n = 10, P)$, if $E(X) = 8$, find the value of P .
- Q.14. Evaluate $\int \sin 4x \cos 3x dx$

Section C

Attempt any Eight of the following questions.

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- Q.15. Examine whether the statement pattern $(p \rightarrow q) \leftrightarrow (\sim p \vee q)$ is a tautology, contradiction or contingency.
- Q.16. Find the inverse of the matrix $\begin{bmatrix} 3 & 2 & 6 \\ 1 & 1 & 2 \\ 2 & 2 & 5 \end{bmatrix}$
- Q.17. Without using truth table, show that $\sim(p \vee q) \vee (\sim p \wedge q) \equiv \sim p$
- Q.18. Show that $\sin^{-1}\left(\frac{8}{17}\right) + \sin^{-1}\left(\frac{3}{5}\right) = \sin^{-1}\left(\frac{77}{85}\right)$
- Q.19. In any $\triangle ABC$, with usual notations, prove that $b^2 = c^2 + a^2 - 2ca \cos B$.
- Q.20. Prove that a homogeneous equation of degree two in x and y (i.e. $ax^2 + 2hxy + by^2 = 0$) represents a pair of lines through origin if $h^2 - ab \geq 0$.
- Q.21. If $e^x + e^y = e^{x+y}$, Show that $\frac{dy}{dx} = -e^{y-x}$
- Q.22. The surface area of a spherical balloon is increasing at the rate of $2\text{cm}^2/\text{sec}$. At what rate the volume of the balloon is increasing when the radius of the balloon is 6 cm.
- Q.23. Find the maximum volume of the cylinder if a right circular cylinder is to be made so that the sum of the radius and height is 6 metres.
- Q.24. Find the mean number of heads in three tosses of a fair coin.

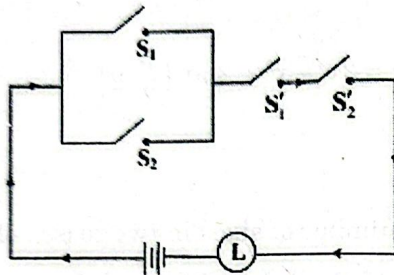
- Q.25. A die is thrown 6 times. If "getting an odd number" is a success. Find the probability of at least 5 successes.
- Q.26. If $x = \Phi(t)$ is a differentiable function of t , then Prove that $\int f(x) dx = \int f[\Phi(t)] \cdot \Phi'(t) dt$

Section D

Attempt any Five of the following questions.

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- Q.27. Find the symbolic form of the given switching circuit. Construct its switching table and interpret your result.



- Q.28. If three numbers are added, their sum is 2. If two times the second number is subtracted from the sum of first and third numbers, we get 8 and if three times the first number is added to the sum of second and third numbers we get 4. Find the numbers using matrices.
- Q.29. In ΔABC , prove that $\tan\left(\frac{C-A}{2}\right) = \left(\frac{c-a}{c+a}\right) \cot \frac{B}{2}$
- Q.30. Solve the L.P.P by graphical method Maximize $Z = 6x + 4y$
Subject to $x \leq 2, x + y \leq 3, -2x + y \leq 1, x, y \geq 0$
Also find the maximum value of Z .
- Q.31. If $y = f(x)$ is a differentiable function of x such that inverse function $x = f^{-1}(y)$ exists, then prove that x is a differentiable function of y and $\left(\frac{dx}{dy}\right) = \frac{1}{\left(\frac{dy}{dx}\right)}$ where $\left(\frac{dy}{dx}\right) \neq 0$. Hence find $\frac{d}{dx}[\sin^{-1}x]$
- Q.32. Solve $\int \frac{2\sin x + 3\cos x}{3\sin x + 4\cos x} dx$
- Q.33. Find K if the function $f(x)$ is defined by $f(x) = Kx(1-x)$, for $0 < x < 1$
 $= 0$ Otherwise
is the probability density function (p.d.f) of a random variable (r.v) X .
Also find $P\left(X < \frac{1}{2}\right)$
- Q.34. Gives $X \sim B(n, P)$, If $n = 20, E(X) = 10$, find $P, \text{Var}(X)$, and $SD(X)$. Also find $P(X > 1)$