CODE : A - M730122 - A	First Semester Examination 2023 - 24	5 x 570
Std. : XII	Sub. : Mathematics & Statistics	Marks : 80
Date : 02/11/2023	Roll no. _____	Time : 3 hrs

General Instructions :-

1. Question paper consist of 34 questions divided into Four sections, namely A,B,C and D.
   - Section - A : Q. No. 1 contains Eight multiple choice type of questions carrying two mark each.
     Q. No. 2 contains four very short answer type of questions carrying One mark each.
   - Section - B : Q. No. 3 to Q. No. 14 contains Twelve short answer type of questions carrying two marks each. (Attempt any Eight)
   - Section - C : Q. No. 15 to Q. No. 26 contains Twelve short answer type of questions carrying Three marks each. (Attempt any Eight)
   - Section - D : Q. No. 27 to Q. No. 34 contains Eight long answer type questions carrying Four mark each. (Attempt any Five)
2. Figures to the right indicate full marks.
3. Start answer to each section on new page.
4. For each MCQ, correct answer must be written along with its alphabet. Eg. a) ..... b) ..... c)..... d) .....
5. Evaluation of each MCQ would be done for the first attempt only.
6. Use of graph is not necessary. Only rough sketch of graph is expected.
7. Use log table if necessary. Use of calculator is not allowed.

Section A

Q.1. Select and write the correct answer for the following multiple choice type of questions : **16**

1) Inverse of the statement pattern $(p \vee q) \longrightarrow (p \wedge q)$ is

a) $(p \wedge q) \longrightarrow (p \vee q)$

b) $\sim (p \vee q) \longrightarrow (p \wedge q)$

c) $(\sim p \vee \sim q) \longrightarrow (\sim p \wedge \sim q)$

d) $(\sim p \wedge \sim q) \longrightarrow (\sim p \vee \sim q)$

2) If $A = \begin{bmatrix} 2 & 3 \\ 5 & -2 \end{bmatrix}$ such that $A^{-1} = KA$, then K equals

a) 19 b) 1/19 c) – 19 d) –1/19

3) The principal solution of $\cos^{-1}\left(-\frac{1}{2}\right)$ is

a) $3\pi/2$ b) $2\pi/3$ c) $\pi/6$ d) $\pi/3$

4) If the distance between lines $(x - 2y)^2 + k(x - 2y) = 0$ is 3 units then K equals.

a) $\pm 3\sqrt{5}$ b) $\pm 5\sqrt{2}$ c) ± 3 d) 0

5) If $x^2 + y^2 = t - \frac{1}{t}$ and $x^4 + y^4 = t^2 + \frac{1}{t^2}$ then $x^3 y \frac{dy}{dx}$ equals.

a) 0 b) –1 c) 2 d) 1

6) $2x^3 - 6x + 5$ is an increasing function, if

a) $0 < x < 1$ b) $x < -1$ or $x > 1$ c) $-1 < x < 1$ d) $-1 < x < -\frac{1}{2}$

7) If the pdf of a continuous random variable is given as

$f(x) = \frac{x^2}{3}$ for $-1 < x < 2$

$= 0$ otherwise

Then c. d. f of X is

a) $\frac{x^3}{9} + \frac{1}{9}$ b) $\frac{x^3}{9} - \frac{1}{9}$ c) $\frac{x^2}{4} + \frac{1}{4}$ d) $\frac{1}{9x^3} + \frac{1}{9}$

8) $\int \sqrt{2+\sqrt{2+\sqrt{2+2\cos 8x}}}\, dx$ equals

a) $\sqrt{2\sin x} + C$ b) $2 \sin x + C$ c) $\sqrt{2}\sin x + C$ d) $2\sqrt{\sin x} + C$

Q.2. Answer the following questions. 4

1) Write the negation of the statement $\exists x \in A$, such that $x + 9 \leq 15$

2) Check whether the following matrix is invertible or not

$$A = \begin{bmatrix} 3 & 4 & 3 \\ 1 & 1 & 0 \\ 1 & 4 & 5 \end{bmatrix}$$

3) Find $\frac{dy}{dx}$ if $y = 16^{\log_4 \tan x}$

4) Find the slope of the normal to the curve $y = a\, e^{\frac{-x}{b}}$ at the point where it crosses the Y axis.

Section B

Attempt any Eight of the following questions. 16

Q.3. Using rules in logic, prove that

$\sim (p \vee q) \vee (\sim p \wedge q) \equiv p$

Q.4. If $A = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$, adj $A = \begin{bmatrix} 4 & a \\ -3 & b \end{bmatrix}$, then Find a and b

Q.5. Find the polar co-ordinates whose cartesian co-ordinates are $(1, -\sqrt{3})$

Q.6. Find the value of k if $2x + y = 0$, is one of the lines represented by $3x^2 + kxy + 2y^2 = 0$.

Q.7. Solve graphically $2x - 3y \geq 0$

Q.8. If $\sin y = x \sin(a + y)$ then show that $\frac{dy}{dx} = \frac{\sin^2(a+y)}{\sin a}$

Q.9. For binomial distribution n = 5 if P (X = 4) = P (X = 3). Find p.

Q.10. Let X represent the difference between the number of heads and number of tails obtained when a coin is tossed 6 times. What are the possible values of X ?

Q.11. Evaluate = $\int \frac{\sqrt{\tan x}}{\sin x \cdot \cos x}\, dx$

Q.12. Find $\frac{dy}{dx}$ if $y = \sin^{-1}\left(\frac{4^{x+\frac{1}{2}}}{1+2^{4x}}\right)$

Q.13. If $A = \begin{bmatrix} x & 0 & 0 \\ 0 & y & 0 \\ 0 & 0 & z \end{bmatrix}$ is a non singular matrix. Find A^{-1}.

Q.14. Solve $\tan^{-1}\left(\frac{1-x}{1+x}\right) = \frac{1}{2}\tan^{-1}x$, $x > 0$

Section C

Attempt any Eight of the following questions. **24**

Q.15. Examine whether the given statement pattern is a tautology or a contradiction or a contingency

$(p \rightarrow q) \leftrightarrow (\sim p \vee q)$

Q.16. If $A = \begin{bmatrix} 2 & -2 \\ 4 & 3 \end{bmatrix}$ find A^{-1} using Adjoint Method.

Q.17. In Δ ABC prove that

$$(b+c-a)\tan\frac{A}{2} = (c+a-b)\tan\frac{B}{2}$$

Q.18. If θ is the measure of acute angle between the pair of lines given by $ax^2 + 2hxy + by^2 = 0$ then prove that $\tan\theta = \left|\frac{2\sqrt{h^2-ab}}{a+b}\right|$, $a + b \neq 0$

Q.19. Write the truth values of the following statements.

a) $\forall$ n $\in$ N, n^2 + n is an even number while n^2 – n is an odd number.

b) $\exists$ n $\in$ N such that n + 5 > 10

c) 4 is odd or 1 is prime.

Q.20. Find the general solution for the equation $\sin\theta = \tan\theta$

Q.21. If y = f (u) is a differentiable function of u and u = g (x) is a differentiable function of x such that the composite function $y = f[g(x)]$ is s differentiable function of x then $\frac{dy}{dx} = \frac{dy}{du} \times \frac{du}{dx}$

Q.22. A window is in the form of a rectangle surrounded by a semi circle. If the perimeter be 30m, find the dimensions so that the greatest possible amount of light may be admitted.

Q.23. It is observed that it rains on 12 days out of 30 days. Find the probability that it will rain on at least 2 days of given week.

Q.24. Let the pmf of random variable X be $P(x) = \frac{3-x}{10}$ for $x = -1, 0, 1, 2$
$= 0$ otherwise.

Calculate E(X) and Var (X)

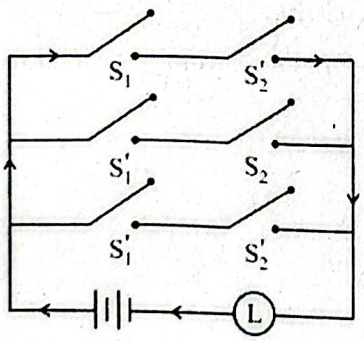
Q.25. Evaluate $\int \frac{1}{2+3\tan x} dx$

Q.26. If Rolles theorem holds for the function $f(x) = x^3 + px^2 + qx + 5$, $x \in [1,3]$ with $c = 2 + \frac{1}{\sqrt{3}}$, find value of p and q.

Section D

Attempt any five of the following questions. **20**

Q.27. Simplify the following circuit so that the new circuit has minimum number of switches. Also draw the simplified circuit.



Q.28. If $A = \begin{bmatrix} 1 & 0 & 1 \\ 0 & 2 & 3 \\ 1 & 2 & 1 \end{bmatrix}$ and $B = \begin{bmatrix} 1 & 2 & 3 \\ 1 & 1 & 5 \\ 2 & 4 & 7 \end{bmatrix}$ then find matrix X such that XA = B.

Q.29. In Δ ABC, prove that $\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C} = 2R$ where R is the circumradius of Δ ABC.

Q.30. Minimize Z = 8x + 10y subject to

$2x + y \geq 7$,
$2x + 3y \geq 15$,
$y \geq 2$,
$x \geq 0, y \geq 0$

Q.31. Find the nth order derivative of $y = e^{8x} \cos(6x + 7)$

Q.32. The following is the cdf of random variable X

X	–3	–2	–1	0	1	2	3	4
F(X)	0.1	0.3	0.5	0.65	0.75	0.85	0.9	1

Find pmf of X

i) $P(-1 \leq X \leq 2)$

ii) $P(X \leq 3 / X > 0)$

Q.33. Find the probability of throwing atleast one four in 6 throws of a single die. Also find Variance.

Q.34. If $x = \phi(t)$ is a differentiable function of t then

$$\int f(x)dx = \int f[\phi(t)]\,\phi^{1}(t)\,dt$$