

Code: A-M337122-A

I Semester Exam – 2022-23**3 x 620**

Std.: XII

Sub: **MATHEMATICS & STATISTICS**

Marks: 80

Date: 15/11/2022

Roll No.: _____

Time: 3 Hrs.

General Instructions :-

- Question paper consists of **34** questions divided into **FOUR** sections, namely **A, B, C** and **D**.
 - Section-A:** Q.No. 1 contains **8 multiple choice** type questions carrying **two marks** each.
Q.No.2 contains **4 very short answers** type questions carrying **one mark** each.
 - Section-B:** Q.No. 3 to Q.No. 14 are **12 short answer** type questions carrying **two marks** each out of which **any eight** are to be attempted.
 - Section-C:** Q.No.15 to Q.No.26 are **12 short answer** type questions carrying **three marks** each out of which **any eight** are to be attempted.
 - Section-D:** Q.No.27 to Q.No.34 are **8 long answer** type questions carrying **four marks** each out of which **any five** are to be attempted.
- Figures to the right indicate full marks.
- Start each section on new page.
- For each MCQ, the correct answer must be written along with its alphabet.
e.g. (a)/ (b)/ (c)/ (d), etc.
- Evaluation of each MCQ would be done for the first attempt only.
- Use of graph paper is not necessary. Only rough sketch of graph is expected.
- Use log table if necessary. Use of calculator is not allowed.

SECTION - A**Q.1 Select and write the most appropriate answer from the given alternatives for each questions :****16**

- $\sim [p \rightarrow (p \wedge \sim q)] \equiv \dots\dots\dots$
 - p
 - q
 - $p \wedge q$
 - $p \vee q$
- If $A = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$ and $A(\text{adj } A) = KI$, then the value of 'K' is
 - 2
 - 2
 - 10
 - 10
- In $\triangle ABC$ if $c^2 + a^2 - b^2 = ac$, then $\angle B = \dots\dots\dots$
 - $\frac{\pi}{4}$
 - $\frac{\pi}{3}$
 - $\frac{\pi}{2}$
 - $\frac{\pi}{6}$
- If the sum of the slopes of the lines represented by $x^2 + kxy - 3y^2 = 0$ is twice their product, then the value of 'K' is
 - 2
 - 1
 - 1
 - 2
- If $f(x) = x^5 + 2x - 3$, then $(f^{-1})^1(-3) = \dots\dots\dots$
 - 0
 - 3
 - $-\frac{1}{3}$
 - $\frac{1}{2}$
- The equation of tangent to the curve $y = 3x^2 - x + 1$ at P (1, 3) is
 - $5x - y = 2$
 - $x + 5y = 16$
 - $5x - y + 2 = 0$
 - $5x = y$
- Let the p.m.f of a random variable X be

$$P(x) = \frac{3-x}{10}, \text{ for } x = -1, 0, 1, 2$$

$$= 0, \text{ otherwise.}$$
 Then E(X) is
 - 1
 - 2
 - 0
 - 1

P.T.O

- viii) Given $X \sim B(n, p)$. If $P = 0.6$ and $E(X) = 6$, then the value of $\text{Var}(X)$ is
 a) 2.4 b) 2.6 c) 2.5 d) 2.3

Q.2 Answer the following questions :

4

- i) Write the dual of $p \wedge \sim p \equiv F$.
 ii) Write the separate equations of lines represented by the equation $5x^2 - 9y^2 = 0$.
 iii) If $y = \sin(\log x)$ then find $\frac{dy}{dx}$
 iv) The velocity V of a particle at time t is given by
 $v = 3t^2 - 8t - 5$. Find acceleration.

SECTION - B

Attempt any EIGHT of the following questions :

16

- Q.3** Write the negations of the following statements.
 a) All students of this college live in the hostel.
 b) 6 is an even number or 36 is a perfect square.
- Q.4** Using truth table verify that $\sim(p \vee q) \equiv \sim p \wedge \sim q$.
- Q.5** In $\triangle ABC$, prove that $ac \cos B - bc \cos A = a^2 - b^2$.
- Q.6** Find the principal solutions of $\cot \theta = 0$.
- Q.7** Solve graphically $x \geq 0, y \geq 0$.
- Q.8** Solve graphically $2x - 3y \geq 0$
- Q.9** If $y = \sqrt{x}$ then find $\frac{d^2y}{dx^2}$
- Q.10** If $y = \tan^2(\log x^3)$, find .
- Q.11** Find the approximate value of $\sqrt{8.95}$.
- Q.12** Check whether the conditions of Rolle's theorem are satisfied by the function
 $f(x) = (x-1)(x-2)(x-3), x \in [1, 3]$
- Q.13** Find the probability distribution of number of heads in two tosses of a coin.
- Q.14** If $X \sim B(n, p)$ and $E(X) = 6$ and $\text{Var}(X) = 4.2$, then find n and p .

SECTION - C

Attempt any EIGHT of the following questions :

24

- Q.15** State the converse, inverse and contrapositive of the conditional statement :
 'If a sequence is bounded, then it is convergent'.
- Q.16** Find the general solution of $\cot x + \tan x = 2 \operatorname{cosec} x$.
- Q.17** Show that $\sin^{-1}\left(\frac{5}{13}\right) + \cos^{-1}\left(\frac{3}{5}\right) = \tan^{-1}\left(\frac{63}{16}\right)$
- Q.18** Find p and q , if the equation $px^2 - 8xy + 3y^2 + 14x + 2y + q = 0$ represents a pair of perpendicular lines.
- Q.19** If θ is the measure of acute angle between the pair of lines represented by
 $ax^2 + 2hxy + by^2 = 0$, then prove that $\tan \theta = \left| \frac{2\sqrt{h^2 - ab}}{a+b} \right|, a+b \neq 0$.
- Q.20** Minimize $Z = 7x + y$ subject to $5x + y \geq 5, x + y \geq 3, x \geq 0, y \geq 0$.
- Q.21** If $x = a \cos^3 t, y = a \sin^3 t$, show that $\frac{dy}{dx} = -\left(\frac{y}{x}\right)^{\frac{1}{3}}$

- Q.22 If $x^7 y^5 = (x + y)^{12}$, then prove that $\frac{dy}{dx} = \frac{y}{x}$.
- Q.23 The surface area of a spherical balloon is increasing at the rate of 2 cm²/sec. At what rate the volume of the balloon is increasing when the radius of the balloon is 6 cm?
- Q.24 Find the approximate value of $e^{1.005}$; given $e = 2.7183$.
- Q.25 The probability mass function for X = number of major defects in a randomly selected appliance of a certain type is

$X = x$	0	1	2	3	4
$P(X = x)$	0.08	0.15	0.45	0.27	0.05

Find the expected value and variance of X .

- Q.26 Suppose that 80% of all families own television set. If 10 families are interviewed at random, find the probability that at most three families own a television set.

SECTION - D

Attempt any FIVE of the following questions :

20

- Q.27 Without using truth table prove that $(p \wedge q) \vee (\sim p \wedge q) \vee (p \wedge \sim q) \equiv (p \vee q)$.
- Q.28 Solve the following equations by the method of reduction :
- $$\begin{aligned} x + 3y + 3z &= 12 \\ x + 4y + 4z &= 15 \\ x + 3y + 4z &= 13 \end{aligned}$$
- Q.29 Solve the following system of equations by the method of inversion :
- $$x - y + z = 4, 2x + y - 3z = 0, x + y + z = 2$$
- Q.30 In $\triangle ABC$ with usual notations, prove that
- $$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C} = 2R$$
- where R is the circum radius of $\triangle ABC$.
- Q.31 If $y = f(x)$ is a differentiable function of x such that inverse function $x = f^{-1}(y)$ exists, then prove that x is a differentiable function of y and $\frac{dx}{dy} = \frac{1}{\frac{dy}{dx}}$, where $\frac{dy}{dx} \neq 0$. Hence find $\frac{d}{dx}(\tan^{-1}x)$
- Q.32 Find the maximum and minimum values of the function $f(x) = x^3 - 9x^2 + 24x$.
- Q.33 Given the probability density function (p.d.f.) of a continuous random variable x as :
- $$f(x) = \frac{x^2}{3}, -1 < x < 2$$
- $$= 0, \text{ otherwise}$$
- Determine the cumulative distribution function (c.d.f) of X and hence find $P(X < 1)$, $P(X > 0)$, $P(1 < X < 2)$.
- Q.34 A fair coin is tossed 8 times. Find the probability that :
- It shows no head.
 - It shows head at least once.