

Code: A-M223122-A

I SEMESTER EXAM - 2024-25

3 x 390

Std.: XII

Sub: Mathematics and Statistics

Marks: 80

Date:

Roll No.: _____

Time:

General Instructions:*The question paper is divided into FOUR sections:*

- (1) **Section A:** Q. No. 1 contains Eight multiple choice type of questions each carrying Two marks.
Q.No. 2 contains Four very short answer type of questions each carrying One mark.
- (2) **Section B:** Q. No. 3 to Q. No. 14 contains Twelve short answer type of questions each carrying Two marks. (Attempt any Eight).
- (3) **Section C:** Q. No. 15 to Q. No. 26 contains Twelve short answer type of questions each carrying Three marks. (Attempt any Eight).
- (4) **Section D:** Q. No. 27 to Q. No. 34 contains Eight long answer type of questions each carrying Four marks. (Attempt any Five).
- (5) Use of log table is allowed. Use of calculator is not allowed.
- (6) Figures to the right indicate full marks.
- (7) Use of graph paper is not necessary. Only rough sketch of graph is expected.

Section A**Q-1** Select and write the correct answer for the following multiple choice type of questions:[16]

- i) If $A = \{1, 2, 3, 4, 5\}$ then which of the following is not true. (2)
 - (a) $\exists x \in A$ such that $x + 3 = 8$
 - (b) $\exists x \in A$ such that $x + 2 < 9$
 - (c) $\forall x \in A$ such that $x + 6 \geq 9$
 - (d) $\exists x \in A$ such that $x + 6 < 10$
- ii) If $A = \begin{bmatrix} -\alpha & 1 \\ -1 & \alpha \end{bmatrix}$ then A^{-1} doesn't exist for $\alpha =$ (2)
 - (a) 0
 - (b) ± 1
 - (c) 2
 - (d) 3
- iii) In any $\triangle ABC$ if $a \cos B = b \cos A$ then the triangle is
 - (a) Equilateral triangle
 - (b) Isosceles triangle
 - (c) Scalene triangle
 - (d) Right angled triangle
- iv) If the distance between the lines $(x - 2y)^2 + k(x - 2y) = 0$ is 3 units, then $k =$ (2)
 - (a) ± 3
 - (b) $\pm 5\sqrt{5}$
 - (c) 0
 - (d) $\pm 3\sqrt{5}$
- v) The function $f(x) = \sin^4 x + \cos^4 x$ is an increasing function if (2)
 - (a) $\frac{\pi}{4} < x < \frac{\pi}{2}$
 - (b) $0 < x < \frac{\pi}{4}$
 - (c) $\frac{\pi}{8} < x < \frac{\pi}{4}$
 - (d) $0 < x < \frac{\pi}{8}$
- vi) If p.d.f. of a random variable X is $f(x) = 6x(1-x)$ for $0 < x < 1$ and 0 otherwise, then the c.d.f. of r.v. $F(x)$ is (2)
 - (a) $2x - 3x^2$
 - (b) $3x - 4x^3$
 - (c) $3x - 2x^3$
 - (d) $2x^3 - 3x$
- vii) The mean and variance of a random variable having a binomial distribution are 4 and 2 respectively, then $P[X = 1]$ is (2)
 - (a) $\frac{1}{32}$
 - (b) $\frac{1}{16}$
 - (c) $\frac{1}{8}$
 - (d) $\frac{1}{4}$
- viii) If the mean and variance of a binomial distribution are 18 and 12 respectively, then $n =$ (2)
 - (a) 36
 - (b) 54
 - (c) 18
 - (d) 27

Q. 2 Answer the following questions :

[4]

- (i) Write the separate equations of the lines $5x^2 - 9y^2 = 0$ (1)
- (ii) Differentiate $\sec^{-1} x$ w.r.t. x (1)

- (iii) The displacement of a particle is given by $S = 5 + 20t - 2t^2$. Find the time when particle stops. (1)
- (iv) Write the type of random variable X where X denotes number of Heads when a coin is tossed 5 times. (1)

Section B

Attempt any EIGHT of the following questions :

[16]

3. Write the negations of the following statements: (2)
- (a) All students of this college live in the hostel
- (b) 6 is an even number or 36 is a perfect square.
4. Find the cofactors of the elements a_{11} and a_{21} of matrix $A = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$ (2)
5. Find the principal solutions of $\cot\theta = 0$ (2)
6. Find the cartesian coordinates of the point whose polar coordinates are $\left(\frac{1}{2}, \frac{\pi}{3}\right)$ (2)
7. If $ax^2 + 2hxy + by^2 = 0$ represents a pair of straight lines and $h^2 - ab \neq 0$ then find the ratio of their slopes. (2)
8. Find k , if the sum of the slopes of the lines represented by $x^2 + kxy - 3y^2 = 0$ is twice their product. (2)
9. Find $\frac{dy}{dx}$ if $y = (\log x)^x$ (2)
10. Differentiate $\log(1 + x^2)$ w.r.t. $\tan^{-1}x$. (2)
11. Show that the function $f(x) = x^3 + 10x + 7$, $x \in \mathbb{R}$ is strictly increasing. (2)
12. If two coins are tossed simultaneously, write the probability distribution of number of Heads. (2)
13. If $X \sim B(n, p)$. If $n = 10$ and $p = 0.4$. Find $E(X)$ and $V(X)$.
14. A fair coin is tossed five times. Find the probability that it shows head exactly three times. (2)

Section C

Attempt any EIGHT of the following questions :

(24)

15. State the converse, inverse and contrapositive of the statement 'If sequence is bounded then it is convergent'. (3)
16. Examine whether the given statement pattern $(\sim p \rightarrow q) \wedge (p \wedge r)$ is a tautology, contradiction or contingency. (3)
17. Find the inverse of the matrix by Adjoint method (3)
- $$A = \begin{bmatrix} 3 & 2 & 6 \\ 1 & 1 & 2 \\ 2 & 2 & 5 \end{bmatrix}$$
18. Prove that $\tan^{-1}\left(\frac{1}{2}\right) + \tan^{-1}\left(\frac{1}{3}\right) = \frac{\pi}{4}$ (3)
19. Prove that the acute angle θ between the pair of straight lines $ax^2 + 2hxy + by^2 = 0$ is given by $\tan\theta = \left| \frac{2\sqrt{h^2 - ab}}{a + b} \right|$, where $a + b \neq 0$. (3)
20. A diet of a sick person must contain at least 48 units of vitamin A and 64 units of vitamin B. (3)
- Two foods F_1 and F_2 are available. Food F_1 costs Rs.6 per unit and food F_2 costs Rs. 10 per unit. One unit of food F_1 contains 6 unit of vitamin A and 7 unit of vitamin B. One unit of food F_2 contains 8 unit of vitamin A and 12 unit of vitamin B. Formulate the LPP so as to minimize the cost.

21. If $y = \sqrt{\tan x + \sqrt{\tan x + \sqrt{\tan x + \dots + \infty}}}$ then
Find $\frac{dy}{dx}$ at $x = 0$. (3)
22. If $y = \sin^{-1}x$, show that $(1 - x^2) \frac{d^2y}{dx^2} - x \frac{dy}{dx} = 0$. (3)
23. Find the approximate value of $\tan^{-1}(1.002)$ given that $\pi = 3.1416$. (3)
24. Find the equation of tangent to the curve $y = 3x^2 - x + 1$ at $P(1,3)$ (3)
25. Find expected value and variance of X , where X is number obtained on the uppermost face when a fair die is thrown. (3)
26. A fair coin is tossed five times. Find the probability that it shows head exactly three times. (3)

Section D

Attempt any FIVE of the following questions :

[20]

27. Without using truth table prove that: $p \leftrightarrow q \equiv (p \wedge q) \vee (\sim p \wedge \sim q)$ (4)
28. Solve the following equations by reduction method (4)
 $x + 3y + 3z = 12$, $x + 4y + 4z = 15$, $x + 3y + 4z = 13$
29. In $\triangle ABC$ with usual notations prove that $b^2 = c^2 + a^2 - 2ca \cdot \cos B$ (4)
30. Solve the following L.P.P. using graphical method. (4)
Maximize: $z = 10x + 25y$
Subject to: $0 \leq x \leq 3$, $0 \leq y \leq 3$, $x + y \leq 5$, $x \geq 0, y \geq 0$.
Also find maximum value.
31. If $y = f(x)$ is a differentiable function of x such that inverse function $x = f^{-1}(y)$ exist, then prove that x is a differentiable function of y and $\frac{dx}{dy} = \frac{1}{\frac{dy}{dx}}$, where $\frac{dy}{dx} \neq 0$. Hence find $\frac{d}{dx}(\sin^{-1}x)$ (4)
32. Verify Lagrange's mean value theorem for $f(x) = \sqrt{x+4}$ on the interval $[0, 5]$. (4)
33. For the following probability density function (p.d.f) of X , (4)
find (i) k (ii) $P\left(x < \frac{1}{2}\right)$, where
$$f(x) = \begin{cases} kx(1-x), & 0 < x < 1 \\ 0, & \text{otherwise} \end{cases}$$
34. It is known that 10% of certain articles manufactured are defective. What is the probability that in a random sample of 12 such articles of 9 articles are defective? (4)